Nuclear shell model studies of exotic nuclei and implications in astrophysics

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Origin of heavy elements

Based on USA National Academy of Science Report

Question 3
How were the elements from iron to uranium made?
Nuclei discussed in this talk
Lanzhou City
CSRe: Cooling storage ring

RIBLL2 + CSRe
Isochronous mass spectrometer

- End of 2007: test run
- End of 2008: 1st physics run
- Fall of 2009: results

\[ \frac{df}{f} = -\frac{1}{\gamma_i^2} \frac{d(m/q)}{(m/q)} + \left(1 - \frac{\gamma_i^2}{\gamma_i^2}ight) \frac{dv}{v} \]

等时性质量谱仪 (IMS): \( \gamma \rightarrow \gamma_i \)
肖特基质量谱仪 (SMS): \( dv \rightarrow 0 \)
These short lived $Z=\text{N}-1$ and $Z=\text{N}-2$ masses have never been measured experimentally. They lie near the proton drip-line and are very exotic nuclei.

These masses so far have been calculated using the best knowledge in nuclear structure models. They can serve as a valuable testing ground.

Masses of particular isotopes are those in connection to the even-even $N=Z$ waiting point nuclei, they are very important for nuclear astrophysical rp process.
Mass measurement results

- Masses of these nuclei are measured for the first time.
- Current HIRFL-CSR results have similar error bars as CDE predictions.
- It confirms that CDE method is reliable at least for $^{63}$Ge, $^{67}$Se.
- It shows some differences for $^{65}$As and $^{71}$Kr.
Difference in binding energy of mirror nuclei

\[ D(A, T) = BE(A, T_<^Z) - BE(A, T_>^Z) \]

Binding energy of the proton-rich nucleus \( BE(A, T_<^Z) \)

Binding energy of the neutron-rich nucleus \( BE(A, T_>^Z) \)

\[ T = |T_<^Z| = |T_>^Z| \]

\( D(A, T) \) calculated with Skyrme Hartreee-Fock method

\[ BE(A, T_<^Z) = D(A, T)_{HF} + BE(A, T_>^Z)_{exp} \]
In these nuclei, different shapes are known to co-exist near the ground state.

Nuclear shape coexistence leads to shape isomeric states (excited states having relatively long lifetimes).

People then questioned the HF method of calculation of CDE.
Energies levels of $^{68}\text{Se}$ and $^{72}\text{Kr}$

Isomer influence on abundances in X-ray burst

- It is possible that a flow towards higher mass through the isomer branch can occur (calculations using multi-mass-zone x-ray burst model)

Without any possible isomer contribution

Full flow through isomers rather than g.s.
Phase transition in exotic nuclei along the $N = Z$ line

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formation properly. Through analyzing the wave functions, we have concluded that the qualitative structure difference between the nuclei with $N = Z \leq 35$ and those with $N = Z \geq 36$ has the origin of the upper "gd" shell occupation. It has been discussed that the structure change is caused by a decisive breaking of the spherical shell model mean field and a formation of deformed mean field. In this sense, we have witnessed a basic mechanism of the phase transition to deformation in nuclei.
rp-process in x-ray burst

Neutron star

rp-process = Rapid proton capture process

One of the major processes for heavy element production

Most of the time is spent at the waiting points
[Diagram of nuclear chart with elements and decay paths]

- **slow β decay** (waiting point)

- **Waiting point nuclei**
89%–90% of the reaction flow passes through $^{64}$Ge via proton capture indicating that:

$^{64}$Ge is not a significant rp-process waiting point.
Direct Mass Measurements of Short-Lived $A = 2Z - 1$ Nuclides $^{63}$Ge, $^{65}$As, $^{67}$Se, and $^{71}$Kr and Their Impact on Nucleosynthesis in the $rp$ Process

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Mass excesses of short-lived $A = 2Z - 1$ nuclei $^{63}$Ge, $^{65}$As, $^{67}$Se, and $^{71}$Kr have been directly measured to be $-46921(37)$, $-46937(85)$, $-46580(67)$, and $-46320(141)$ keV, respectively. The deduced proton separation energy of $-90(85)$ keV for $^{65}$As shows that this nucleus is only slightly proton unbound. X-ray burst model calculations with the new mass excess of $^{65}$As suggest that the majority of the reaction flow passes through $^{64}$Ge via proton capture, indicating that $^{64}$Ge is not a significant $rp$-process waiting point.

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NUCLEAR ASTROPHYSICS

Star bursts pinned down

One of the main uncertainties in the burn-up of X-ray bursts from neutron stars has been removed with the weighing of a key nucleus, $^{65}$As, at a new ion storage ring.

Philip Walker

Understanding how the chemical elements formed in stars, and how their formation is related to observable astrophysical phenomena, requires close cooperation between those in the stellar environment. It sounds straightforward, but when the object of attention is $^{65}$As (containing 33 protons and 32 neutrons) with a half-life of only 130 ms, nothing should be taken for granted. As they report in Physical Review Letters, Tu et al.\(^1\) —

The core of the Lanzhou facility is the experimental cooler storage ring (Fig. 1), which is part of an accelerator complex at the Institute of Modern Physics. It is only the second storage ring of its kind in the world, the first being at the Helmholtzzentrum für Schwerionenforschung in Darmstadt, Germany\(^3\). There has been close cooperation between the two centres, and now the Chinese facility has produced its first high-profile results\(^1\).
Important factors for a nuclear structure model

- **Single particle states (mean field part)**
  - Reflect shell structure (spherical, deformed)
  - Adjust to experiment

- **Two-body interactions (residual part)**
  - Mix configurations (do not have in mean field models)
  - Transition probabilities are sensitive test

- **Model space (configurations)**
  - Large enough to cover important parts of physics
  - If not possible, introduce effective parameters
Nuclear structure models

- Shell-model diagonalization method
  - Most fundamental, quantum mechanical ☺
  - Growing computer power helps extending applications ☺
  - A single configuration contains no physics
  - Huge basis dimension required, severe limit in applications

- Mean-field method
  - Applicable to any size of systems ☺
  - Fruitful physics around minima of energy surfaces ☺
  - No configuration mixing
  - States with broken symmetry, cannot be used to calculate electromagnetic transitions and decay rates
Bridge between shell-model and mean-field method

- Projected shell model
  - Use more physical states (e.g. solutions of a deformed mean-field) and angular momentum projection technique to build shell model basis
  - Perform configuration mixing (a shell-model concept)

- The method works in *between* conventional shell model and mean field method, hopefully take the advantages of *both*
General structure features for neutron-rich Cr and Fe nuclei

- largest deformation at $N=38$ or 40, mid-shell effect
  - Smallest $2^+$ excitation, strongest $B(E2)$
  - Ni, Zn, Ge isotopes do not show similar trend
- Important neutron $g_{9/2}$ physics
  - Back-bending along Yrast line
  - Negative-parity bands
  - $I=9/2$ band in odd-mass isotopes
- Softness near ground state
  - No well-defined shape in ground state
  - Possibility of prolate-oblate shape competition near ground state
  - Shape stabilized when nuclei rotate ($I \sim 6$)
Neutron-rich Fe isotopes: Projected shell model calculations

Rotational properties of neutron-rich Fe isotopes

- Comparison of calculated moments of inertia with data
  - Irregularity at $I \sim 8$: alignment of $g_{9/2}$ neutrons
  - at $I \sim 16$: alignment of $f_{7/2}$ protons

![Graph showing moments of inertia for Fe isotopes](image)
Large-scale shell model with only fp shell space cannot describe small $E(2^+)$ energy near $N=40$.

Projected shell model including the neutron $g_{9/2}$ orbit describes the data correctly.
Wavefunctions with and without $g_{9/2}$ orbit?

- Shell models in smaller bases may reproduce energy levels, but the wave functions (B(E2)'s) can be wrong.


\[ B(E2, 2^+\rightarrow 0^+) = 214(26) \text{ e}^2 \text{ fm}^4 \] for N=36

\[ B(E2, 2^+\rightarrow 0^+) = 470 (210) \text{ e}^2 \text{ fm}^4 \] for N=38
The role of neutron $g_{9/2}$ intruder

- The best place to study the role of neutron $g_{9/2}$ orbit is from $I^\pi = 9/2^+$ state (band) in odd-neutron nuclei

R. Ferrer et al. PRC 81, 044318 (2010)
Explore the nature of $9/2^+$ band

- Does the observed $9/2^+$ isomer in odd-mass Fe and Cr nuclei have a $K = 9/2$ component of neutron $g_{9/2}$ state with prolate deformation? **Not possible**
- Or a $K = 9/2$ component of neutron $g_{9/2}$ state with oblate deformation? ($^{59}\text{Cr}$)
Possible shape of $9/2^+$ band

Table 1
For the $9/2^+$ state in $^{59}$Cr: The experimental energy level [15] and the spherical shell model calculation (both in MeV), calculated spectroscopic quadrupole moment $Q_s$ (in $e$ fm$^2$), and deduced quadrupole deformation $\beta$ by assuming $K = 1/2$ or $9/2$.

<table>
<thead>
<tr>
<th>$I^\pi$</th>
<th>$E_x$(Exp)</th>
<th>$E_x$(SM)</th>
<th>$Q_s$(SM)</th>
<th>$\beta$ ($K = 1/2$)</th>
<th>$\beta$ ($K = 9/2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9/2^+$</td>
<td>0.503</td>
<td>0.490</td>
<td>-39.44</td>
<td>+0.273</td>
<td>-0.182</td>
</tr>
</tbody>
</table>

Nature of $9/2^+$ band in odd-mass Cr and Fe isotopes

- PSM calculation shows it has a prolate deformation, mainly of $K=1/2$ component of neutron $g_{9/2}$ state
- An intruder large $j$ orbit with small $K$ component ($K=1/2$) is a strongly decoupled state which shows decoupling effect
- Bandhead has a larger $I$
- Low-spin members lie higher
- Only a favored branch is observed
Comparison of $9/2^+$ band in $^{59}$Cr with prolate and oblate deformation

Spectroscopy of valence nuclei with one particle in empty shells or one hole in completely filled shells provides direct information on single-particle structure.

Spectroscopy of nuclei with two particles or two holes provides information on *correlations* between different kinds of pairs.
Previous shell model studies

- Spectra consist of two groups of excitations:
  - Excitations of pure valence single particles (lower energy)
  - Excitations of valence single particles coupled with core excitation (higher energy)

- Calculations of “simple” shell models
  - Lower energy spectra calculated by shell models with effective interaction of CD Bonn potential
    (the Oslo group, M. Hjorth-Jensen et al.
     the Italy group, A. Covello et al.
     the MSU group, A. Brown et al.)
  - Higher energy spectra described by empirical interactions
    (J. Blomqvist et al.)
Our shell model calculation

- Shell model space:
  
  5 proton orbits \((0g_{7/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 0h_{11/2})\),
  5 upper neutron orbits \((1f_{7/2}, 2p_{3/2}, 0h_{9/2}, 2p_{1/2}, 1f_{5/2})\),
  2 lower neutron orbits \((0h_{11/2}, 1d_{3/2})\).

- Hamiltonian: Extended pairing plus quadrupole-quadrupole with monopole corrections

\[
H = H_{sp} + H_{P_0} + H_{P_2} + H_{QQ} + H_{OO} + H_{HH} + H_{mc}
\]

- Parameters:
  - experimental single-particle states
  - \(5 \times 3 = 15\) two-body parameters
  - 6 monopole corrections
Shell model calculation considering particle-hole excitations show features supporting magnetic rotation bands.
The first mass experiment at Lanzhou CSRe successfully measured several short lived Z=N-1 and Z=N-2 nuclei near the proton drip-line.

The measured masses test existing nuclear structure models that calculate nuclear masses.

Using the new mass of $^{65}$As, $^{64}$Ge is suggested not to be a significant waiting point nucleus.

Projected shell model describes the large deformation region of neutron-rich Cr and Fe nuclei with N~40.

Large-scale spherical shell model with neutron core excitation describes nuclei beyond $^{132}$Sn.
Model space constructed by angular-momentum projected states

- Wavefunction: \[ \psi^I_M = \sum_{\kappa} f_{\kappa} \hat{P}^I_{MK\kappa} |\phi_{\kappa}\rangle \]
  with a.-m.-projector: \[ \hat{P}^I_{MK\kappa} = \frac{2I + 1}{8\pi^2} \int d\Omega \, D^I_{MK\kappa} (\Omega) \tilde{D}(\Omega) \]

- Eigenvalue equation: \[ \sum_{\kappa'} \left( H^I_{\kappa\kappa'} - EN^I_{\kappa\kappa'} \right) f_{\kappa'} = 0 \]
  with matrix elements: \[ H^I_{\kappa\kappa'} = \langle \phi_{\kappa} | \hat{H} \hat{P}^I_{\kappa\kappa'} |\phi_{\kappa}\rangle \quad N^I_{\kappa\kappa'} = \langle \phi_{\kappa} | \hat{P}^I_{\kappa\kappa'} |\phi_{\kappa}\rangle \]

- Hamiltonian is diagonalized in the projected basis
  \[ \left\{ \hat{P}^I_{MK\kappa} |\phi_{\kappa}\rangle \right\} \]
Hamiltonian and single particle space

- The Hamiltonian
  \[ H = H_0 - \sum_{\lambda} \frac{\kappa_\lambda}{2} \sum_{\mu} Q_{\lambda\mu}^+ Q_{\lambda\mu} - G_M P^+ P - G_Q \sum_{\mu} P_{\mu}^+ P_{\mu} \]

- Interaction strengths
  - \( \kappa \) is related to deformation \( \varepsilon \) by
    \[ \kappa_{\tau\tau'} = \frac{2/3}{\hbar \omega_\tau \hbar \omega_{\tau'}} \frac{\varepsilon \hbar \omega_\tau \hbar \omega_{\tau'}}{\hbar \omega_n \langle Q_0 \rangle_n + \hbar \omega_p \langle Q_0 \rangle_p} \]
  - \( G_M \) is fitted by reproducing moments of inertia
  - \( G_Q \) is assumed to be proportional to \( G_M \) with a ratio \( \sim 0.15 \)

- Single particle space
  - Three major shells for neutrons or protons
    For very heavy nuclei, \( N = 5, 6, 7 \) for neutrons
    \( N = 4, 5, 6 \) for protons
Building blocks: a.-m.-projected multi-quasi-particle states

- Even-even nuclei:
  \[ \{ \hat{P}^I_{MK} \alpha_+^+ \alpha_+^+ | 0 \}, \hat{P}^I_{MK} \alpha_\pi^+ \alpha_\pi^+ | 0 \}, \hat{P}^I_{MK} \alpha^+_v \alpha^+_v \alpha^+_v \alpha^+_v | 0 \}, \ldots \} \]

- Odd-odd nuclei:
  \[ \{ \hat{P}^I_{MK} \alpha_+^+ \alpha_\pi^+ | 0 \}, \hat{P}^I_{MK} \alpha_\pi^+ \alpha_\pi^+ \alpha_\pi^+ | 0 \}, \hat{P}^I_{MK} \alpha^+_v \alpha^+_v \alpha^+_v \alpha^+_v \alpha^+_v | 0 \}, \ldots \} \]

- Odd-neutron nuclei:
  \[ \{ \hat{P}^I_{MK} \alpha_\pi^+ | 0 \}, \hat{P}^I_{MK} \alpha_\pi^+ \alpha_\pi^+ \alpha_\pi^+ | 0 \}, \hat{P}^I_{MK} \alpha^+_v \alpha^+_v \alpha^+_v \alpha^+_v \alpha^+_v | 0 \}, \ldots \} \]

- Odd-proton nuclei:
  \[ \{ \hat{P}^I_{MK} \alpha_\pi^+ | 0 \}, \hat{P}^I_{MK} \alpha_\pi^+ \alpha_\pi^+ \alpha_\pi^+ | 0 \}, \hat{P}^I_{MK} \alpha^+_v \alpha^+_v \alpha^+_v \alpha^+_v \alpha^+_v | 0 \}, \ldots \} \]